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A near-wall two-equation heat transfer model for wall turbulent flows

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Abstract

A proposed near-wall $\overline{t^2}$–ε_t two-equation model for turbulent heat transport reproduces the correct near-wall behavior of temperature under various wall thermal boundary conditions. In this model, a mixing timescale is introduced to model the production term of ε_t equation, and a more convenient boundary condition for ε_t under the uniform wall heat flux is suggested. The present model is tested through application to turbulent heat transfer for channel flow. Predicted results are compared with direct numerical simulation (DNS) data. The near-wall $\overline{t^2}$–ε_t two-equation model predicts reasonably well the distributions of the time-mean temperature, normal turbulent heat flux, temperature variance, dissipation rate and their near-wall budgets.

Keywords: Two-equation model; Turbulent flow; Heat transfer

1. Introduction

Numerical prediction of turbulent heat transfer phenomena has attracted substantial interest over the past few decades. A set of differential equations for the Reynolds stress ($-\overline{u_i u_j}$) and turbulent heat flux ($-\overline{u_i t}$) should be solved simultaneously for the time-mean velocity and temperature.

The k–ε turbulence model for velocity field is widely used in engineering applications [1–4]. As for scalar turbulence, the conventional method is the zero-equation heat transfer model, in which the eddy diffusivity for heat α_t is obtained by $\alpha_t = \nu_t / Pr_t$ with the turbulent Prandtl number Pr_t as a constant. However, no universal value of Pr_t was found. Nagano and Kim [5] developed a two-equation model for the thermal field, in which eddy diffusivity for heat α_t was modeled using the temperature variance $\overline{t^2}$ and its dissipation rate ε_t, together with k and ε. Youssef et al. [6] modified the NK model to determine the wall-limiting behavior of turbulence quantities under various wall thermal conditions. However, it is not so convenient to calculate complex heat transfer problems with uniform wall heat flux. Consequently, further improvement of their model would be needed.

In the present study, we will develop a new near-wall $\overline{t^2}$–ε_t model. Using the near-wall behavior of turbulence quantities, we modify the modeling of the production term in ε_t equation, and propose a new boundary condition for ε_t under uniform wall heat flux, which is more convenient for complex heat transfer, as compared with that of Youssef et al. [6]. Both the low-Reynolds-number k–ε turbulence model [3] and our proposed two-equation heat transfer model

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Nomenclature

c_p	specific heat at constant pressure	U_i, u_i	mean and fluctuating velocity, respectively
C_λ, C_{p1}, C_{p2}, C_{d1}, C_{d2}	turbulence model constants for the temperature field	x, y	coordinates in streamwise and wall normal directions
C_μ, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$	turbulence model constants for the velocity field	$-\overline{u_i u_j}$	Reynolds stress tensor
		$-\overline{u_i t}$	Reynolds heat flux vector
f_μ, f_ε	turbulence model functions for the velocity field	y^+	dimensionless distance from wall, $u_\tau y/\nu$
f_λ, f_{d1}, f_{d2}	turbulence model functions for the temperature field		
		Greek symbols	
k	turbulence kinetic energy	α, α_t	molecular and turbulent thermal diffusivities, respectively
p	mean pressure		
Pr, Pr_t	molecular and turbulent Prandtl number, respectively	δ_{ij}	Kronecker delta
q_w	wall heat flux	ε	dissipation rate of k
R	timescale ratio, τ_t/τ_u	ε_t	dissipation rate of $\overline{t^2}$
R_t	turbulent Reynolds number, $k^2/\nu\varepsilon$	ν, ν_t	molecular and turbulent viscosities, respectively
R_ε	turbulent Reynolds number, $y(\nu\varepsilon)^{1/4}/\nu$	σ_k, σ_ε, σ_h, σ_ϕ	turbulence model constants for diffusion of k, ε, $\overline{t^2}$, ε_t
t_τ	friction temperature, $q_w/\rho c_p u_\tau$	τ	time
		τ_w	wall shear stress
$\overline{t^2}$	temperature variance	τ_m	mixing timescale
T, t	mean and fluctuating temperature, respectively	τ_u, τ_t	timescales of velocity and temperature
T^+	dimensionless temperature, $(T_w - T)/t_\tau$		
		Subscripts	
u_τ	friction velocity, $(\tau_w/\rho)^{1/2}$	w	wall

were applied to study convective heat transfer for a two-dimensional turbulent channel flow.

2. Two-equation model for velocity field

The governing equations for an incompressible velocity field with a low-Reynolds-number k–ε model can be written as:

$$\frac{\partial U_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{DU_i}{D\tau} = -\frac{1}{\rho}\frac{\partial P}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\nu \frac{\partial U_i}{\partial x_j} - \overline{u_i u_j}\right) \tag{2}$$

$$\frac{Dk}{D\tau} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] - \overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - \varepsilon \tag{3}$$

$$\frac{D\varepsilon}{D\tau} = \frac{\partial}{\partial x_j}\left[\left(\nu + \frac{\nu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right] - C_{\varepsilon 1}\frac{\varepsilon}{k}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} - C_{\varepsilon 2} f_\varepsilon \frac{\varepsilon^2}{k} \tag{4}$$

$$\nu_t = C_\mu f_\mu \frac{k^2}{\varepsilon} \tag{5}$$

$$-\overline{u_i u_j} = -\frac{2}{3}k\delta_{ij} + 2\nu_t S_{ij} \tag{6}$$

In the present study, the low-Reynolds-number k–ε model by Abe et al. [3] is adopted, i.e. the model functions and constants are as follows:

$$f_\mu = \left[1 - \exp\left(-\frac{R_\varepsilon}{14}\right)\right]^2 \left[1 + \frac{5}{R_t^{3/4}}\exp\left\{-\left(\frac{R_t}{200}\right)^2\right\}\right]$$

$$f_\varepsilon = \left\{1 - \exp\left(-\frac{R_\varepsilon}{3.1}\right)\right\}^2 \left\{1 - 0.3\exp\left[-\left(\frac{R_t}{6.5}\right)^2\right]\right\}$$

$$C_\mu = 0.09, \quad \sigma_k = 1.4, \quad \sigma_\varepsilon = 1.4$$

$$C_{\varepsilon 1} = 1.5, \quad C_{\varepsilon 2} = 1.9 \tag{7}$$

3. Formulation of a two-equation model for the thermal field

3.1. Modeling of temperature variance and its dissipation rate equations

The governing equations for turbulent heat transport are expressed as:

$$\frac{DT}{D\tau} = \frac{\partial}{\partial x_i}\left[\alpha \frac{\partial T}{\partial x_i} - \overline{u_i t}\right] \tag{8}$$

$$-\overline{u_i t} = \alpha_t \frac{\partial T}{\partial x_i} \tag{9}$$

$$\alpha_t = C_\lambda f_\lambda k\left(\frac{k}{\varepsilon}\right)^n\left(\frac{\overline{t^2}}{\varepsilon_t}\right)^m \quad n+m=1 \tag{10}$$

Here, the eddy diffusivity for heat α_t is associated with temperature variance, $\overline{t^2}$, and its dissipation rate, ε_t. C_λ denotes the model constant and f_λ is the model function including the near-wall effect in a thermal field, to be described later.

The exact governing equations of $\overline{t^2}$ and ε_t are given by [7]

$$\frac{D\overline{t^2}}{D\tau} = \frac{\partial}{\partial x_j}\left[\alpha \frac{\partial \overline{t^2}}{\partial x_j} - \overline{u_j t^2}\right] - \overline{u_j t}\frac{\partial T}{\partial x_j} - 2\varepsilon_t \tag{11}$$

$$\begin{aligned}\frac{D\varepsilon_t}{D\tau} = {} & \frac{\partial}{\partial x_j}\left(\alpha \frac{\partial \varepsilon_t}{\partial x_j} - \overline{u_j \varepsilon_t'}\right) - 2\alpha \frac{\overline{\partial u_j \partial t \partial t}}{\partial x_k \partial x_k \partial x_j} \\ & - 2\alpha \frac{\overline{\partial u_j \partial t}\partial T}{\partial x_k \partial x_k \partial x_j} - 2\alpha \frac{\overline{\partial t \partial t}\partial U_j}{\partial x_j \partial x_k \partial x_k} \\ & - 2\alpha u_j \frac{\overline{\partial t}\partial^2 T}{\partial x_k \partial x_j \partial x_k} - 2\overline{\left(\alpha \frac{\partial^2 t}{\partial x_j \partial x_k}\right)^2}\end{aligned} \tag{12}$$

In Eqs. (11) and (12), the turbulent diffusion terms $\overline{u_j t^2}$ and $\overline{u_j \varepsilon_t'}$ are approximated, respectively, as [5,6]

$$-\overline{u_j t^2} = \frac{\alpha_t}{\sigma_h}\frac{\partial \overline{t^2}}{\partial x_j} \tag{13}$$

$$-\overline{u_j \varepsilon_t'} = \frac{\alpha_t}{\sigma_\phi}\frac{\partial \varepsilon_t}{\partial x_j} \tag{14}$$

We follow Youssef et al. [6] to take simply the model constants σ_h and σ_ϕ both equal to 1.0.

The production term in Eq. (12) is given as

$$\begin{aligned}P_{\varepsilon_t} = {} & -2\alpha \frac{\overline{\partial u_j \partial t \partial t}}{\partial x_k \partial x_k \partial x_j} - 2\alpha \frac{\overline{\partial u_j \partial t}\partial T}{\partial x_k \partial x_k \partial x_j} \\ & - 2\alpha \frac{\overline{\partial t \partial t}\partial U_j}{\partial x_j \partial x_k \partial x_k} - 2\alpha u_j \frac{\overline{\partial t}\partial^2 T}{\partial x_k \partial x_j \partial x_k}\end{aligned} \tag{15}$$

It has more time and generation-rate scales to be determined [9,10]. Jones and Mansonge [11] and Nagano and Kim [4] try to take the following form

$$P_{\varepsilon_t} = -C_{p2}\frac{\varepsilon}{k}\overline{u_i t}\frac{\partial T}{\partial x_i} - C_{p3}\frac{\varepsilon_t}{k}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j} \tag{16}$$

and

$$P_{\varepsilon_t} = -C_{p1}\frac{\varepsilon_t}{\overline{t^2}}\overline{u_i t}\frac{\partial T}{\partial x_i} - C_{p3}\frac{\varepsilon_t}{k}\overline{u_i u_j}\frac{\partial U_i}{\partial x_j}, \tag{17}$$

respectively. Then, both the production rates for velocity and temperature are used. Newman et al. [12] used only the thermal production rate and thermal timescale. Elgobashi and Launder [13] expressed it as

$$P_{\varepsilon_t} = -C_{p1}\frac{\varepsilon_t}{\overline{t^2}}\overline{u_i t}\frac{\partial T}{\partial x_i} \tag{18}$$

In the present study, instead of Eq. (15), a mixing timescale and only the thermal production rate are used to model the production term in Eq. (12) as

$$P_{\varepsilon_t} = -C_{p1}\left(\frac{\varepsilon\varepsilon_t}{k\overline{t^2}}\right)^{1/2}\overline{u_i t}\frac{\partial T}{\partial x_i} \tag{19}$$

The last term on the right hand side of Eq. (12) stands for destruction due to fine-scale turbulence interaction, which is dependent on both the velocity and thermal timescales. Launder [7] suggested using two terms proportional to $\varepsilon\varepsilon_t/k$ and $\varepsilon_t^2/\overline{t^2}$ [7], which are adopted by many investigators [11–14] also. It reads

$$\varepsilon_{\varepsilon_t} = -C_{d1}f_{d1}\frac{\varepsilon_t^2}{\overline{t^2}} - C_{d2}f_{d2}\frac{\varepsilon_t\varepsilon}{k} \tag{20}$$

The modeled governing equations can be finally described as

$$\frac{D\overline{t^2}}{D\tau} = \frac{\partial}{\partial x_j}\left[\left(\alpha + \frac{\alpha_t}{\sigma_h}\right)\frac{\partial \overline{t^2}}{\partial x_j}\right] - 2\overline{u_j t}\frac{\partial T}{\partial x_j} - 2\varepsilon_t \tag{21}$$

$$\frac{D\varepsilon_t}{D\tau} = \frac{\partial}{\partial x_j}\left[\left(\alpha + \frac{\alpha_t}{\sigma_\phi}\right)\frac{\partial \varepsilon_t}{\partial x_j}\right] - C_{p1}\sqrt{\frac{\varepsilon\varepsilon_t}{k\overline{t^2}}}\,\overline{u_j t}\,\frac{\partial T}{\partial x_j} - C_{d1} f_{d1}\frac{\varepsilon_t^2}{\overline{t^2}} - C_{d2} f_{d2}\frac{\varepsilon\varepsilon_t}{k} \tag{22}$$

3.2. Near-wall modeling and model constants

Using the thermal–mechanical timescale ratio $R = \tau_t/\tau_u$, Eq. (10) may be rewritten as

$$\alpha_t = C_\lambda f_\lambda \frac{k^2}{\varepsilon}(2R)^m \tag{23}$$

Considering that the ratio between the temperature- and velocity-timescales for dissipative motions is represented by $(R/Pr)^{1/2}$ [15], the following relation holds in the vicinity of the wall [8]

$$\nu_t/\alpha_t \propto R^{1/2}. \tag{24}$$

Taking into account f_μ, we write f_λ as

$$f_\lambda = \left[1 - \exp\left(-\frac{R_\varepsilon}{A_\lambda}\right)\right]^2\left[1 + \frac{B_\lambda}{R_t^{3/4}}\frac{(2R)^{0.5}}{(2R)^m}\right] \tag{25}$$

The near-wall behavior of turbulent quantities can be determined using a Taylor series expansion with respect to y:

$$U = A_1 y + A_2 y^2 + A_3 y^3 + \cdots$$

$$u = a_1 y + a_2 y^2 + a_3 y^3 + \cdots$$

$$v = b_2 y^2 + b_3 y^3 + \cdots$$

$$w = c_1 y + c_2 y^2 + c_3 y^3 + \cdots$$

$$k = \frac{1}{2}\left(a_1^2 + c_1^2\right)y^2 + \cdots$$

$$\overline{uv} = \overline{a_1 b_2} y^3 + \left(\overline{a_1 b_3} + \overline{a_2 b_2}\right)y^4 + \cdots$$

$$\varepsilon_w = \nu\left(\frac{\partial^2 k}{\partial y^2}\right)_w = \nu\left(a_1^2 + c_1^2\right)$$

$$T^+ = Pr y^+ + e_1 y^{+2} + \cdots$$

$$t = d_1 y + d_2 y^2 + \cdots$$

$$\overline{t^2} = \overline{d_1^2} y^2 + \cdots$$

$$\overline{vt} = \overline{b_2 d_1} y^3 + \cdots \quad \text{(under uniform wall temperature)}$$

$$t = t_w + d_2 y^2 + \cdots$$

$$\overline{t^2} = \overline{t_w^2} + 2\overline{d_2 t_w} y^2 + \cdots$$

$$\overline{vt} = \overline{b_2 t_w} y^2 + \overline{b_3 t_w} y^3 + \cdots \quad \text{(under uniform wall heat flux)}$$

$$\varepsilon_{tw} = \alpha\left(\frac{\partial^2 \overline{t^2}}{\partial y^2}\right)_w = \frac{\alpha}{2}\left(\overline{d_1^2} + 2\overline{d_2 t_w}\right) \tag{26}$$

From Eq. (26), in the vicinity of the wall, $\overline{vt} \propto y^3$ (under uniform wall temperature) $\overline{vt} \propto y^2$ (under uniform wall heat flux) must be satisfied. Submitting Eqs. (23), (25) and (26) into Eq. (9), the turbulent heat flux $\overline{vt}$, obtained from Eq. (25), satisfies the near-wall limiting behavior under the thermal boundary conditions with and without wall temperature fluctuations, regardless of the mixing timescale τ_m. Hence, it is convenient to use different τ_m in Eq. (25). For the sake of simplicity, we choose $\tau_m = \tau_u^{0.5}(2\tau_t)^{0.5}$, as used by Sommer et al. [9]. Thus,

$$f_\lambda = \left[1 - \exp\left(-\frac{R_\varepsilon}{16}\right)\right]^2\left[1 + \frac{3}{R_t^{3/4}}\right] \tag{27}$$

and

$$\alpha_t = C_\lambda f_\lambda \frac{k^2}{\varepsilon}(2R)^{0.5} \tag{28}$$

The direct numerical simulation (DNS) data [16,17] indicates that the molecular diffusion term balances with the dissipation term at the wall in ε_t equation. From Eq. (11), we have at $y = 0$:

$$\alpha\frac{\partial^2 \varepsilon_t}{\partial y^2} = C_{d1} f_{d1}\frac{\varepsilon_t^2}{\overline{t^2}} + C_{d2} f_{d2}\frac{\varepsilon\varepsilon_t}{k} \tag{29}$$

Considering the limiting behavior of wall turbulence, $f_{d2} \propto y^2$ and $f_{d1} \propto y^2$ (under uniform wall temperature) of $f_{d1} \propto y^n$ with $n > 0$ (under uniform wall heat flux) need to satisfy Eq. (29), the following equations are proposed to meet the requirements.

$$f_{d1} = \left[1 - \exp\left(-\frac{R_\varepsilon}{1.7}\right)^2\right] \tag{30}$$

$$f_{d2} = \left(\frac{1}{C_{d2}}\right)(C_{\varepsilon 2} f_\varepsilon - 1)\left[1 - \exp\left(-\frac{R_\varepsilon}{5.8}\right)^2\right] \tag{31}$$

with $f_\varepsilon = 1 - 0.3\exp\{-(R_t/6.5)^2$.

The model constants can be, therefore, determined in the following way.

First, C_λ, C_{d1}, C_{d2}, σ_h and σ_ϕ are set to 0.1, 2.0, 0.9, 1.0, and 1.0, respectively, following the lead of the model by Youssef et al. [6]. The constant C_{p1} is to be determined with the relation for the constant-stress and constant-heat-flux layer [5,6]:

$$C_{p1} = \frac{C_{d1}}{\sqrt{2R}} + C_{d2} - \frac{(\kappa^2/Pr_t)}{\sigma_\phi\sqrt{C_\mu}} \tag{32}$$

Then, $C_{p1} = 2.34$ by substituting the standard value of C_μ, C_{d1}, C_{d2}, $Pr_t = 0.9$, σ_ϕ and $\kappa = 0.39$–0.41.

In summary, the model constants in the present model are given as follows:

$$C_\lambda = 0.1, \qquad C_{d1} = 2.0, \qquad C_{d2} = 0.9$$

$$C_{p1} = 2.34, \qquad \sigma_h = 1.0, \qquad \sigma_\phi = 1.0 \tag{33}$$

4. Numerical scheme and boundary conditions

The governing equations are discretized by means of the finite volume. The QUICK scheme is used for the approximation of convection terms and central difference for diffusion terms, with SIMPLEC [18] algorithm to handle pressure–velocity coupling. The set of discretized linear algebraic equations is solved by ADI.

The boundary conditions at the wall ($y = 0$) for a velocity field are $U = V = k = 0$ and $\varepsilon_w = \nu\partial^2 k/\partial y^2$ or equivalently $\varepsilon_w = 2\nu(\partial\sqrt{k}/\partial y)^2$. The wall thermal boundary conditions are taken as:

$$T = T_w \qquad \overline{t^2} = 0 \tag{34a}$$

for uniform wall temperature,

$$-\frac{\partial T}{\partial y} = q_w \qquad \frac{\partial \overline{t^2}}{\partial y} = 0 \tag{34b}$$

for uniform wall heat flux, and

$$-\frac{\partial T}{\partial y} = 0 \qquad \frac{\partial \overline{t^2}}{\partial y} = 0 \tag{34c}$$

for adiabatic wall.

We derive the boundary condition for ε_t according to different wall thermal conditions. For uniform wall temperature, from Eq. (26), a fluctuating temperature near the wall can be expressed as

$$t = d_1 y + d_2 y^2 + \cdots$$

$$\overline{t^2} = \overline{d_1^2} y^2 + \cdots \tag{35}$$

The actual ε_t at the wall may be given by

$$\varepsilon_{tw} = \alpha\overline{\left(\frac{\partial t}{\partial y}\right)^2}\Bigg|_w = \alpha\overline{d_1^2} \tag{36}$$

That is,

$$\alpha\left(\frac{\partial\sqrt{\overline{t^2}}}{\partial y}\right)^2\Bigg| = \alpha\overline{d_1^2}$$

So, we can obtain $\varepsilon_{tw} = \alpha(\partial\sqrt{\overline{t^2}}/\partial y)^2_w$ for the case of uniform wall temperature.

For uniform wall heat flux and adiabatic wall, wall heat flux would be a constant or 0. Hence, from Eq. (26), the near wall behavior of fluctuating temperature will be

$$t = t_w + d_2 y^2 + \cdots \tag{37}$$

Substituting Eq. (37) into the definition of ε_t:$\varepsilon_t = \alpha\overline{(\partial t/\partial x_i)^2}$, we obtain

$$\varepsilon_t = \alpha\overline{\left(\frac{\partial t_w}{\partial x}\right)^2} + \alpha\left[2\overline{\left(\frac{\partial t_w \partial d_2}{\partial x \partial x}\right)} + 4\overline{d_2^2}\right]y^2 + \ldots \tag{38}$$

In Eq. (38), no linear term exists. So, we can use the following relation as the boundary condition for ε_t under a uniform wall heat flux and an adiabatic wall:

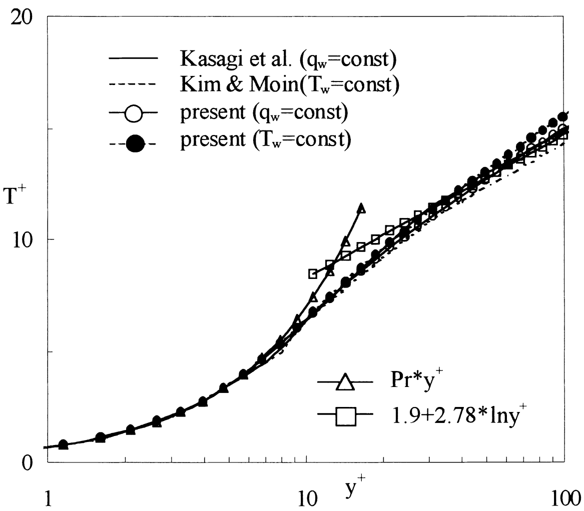

Fig. 1. Mean temperature for the case of constant wall heat flux and constant wall temperature.

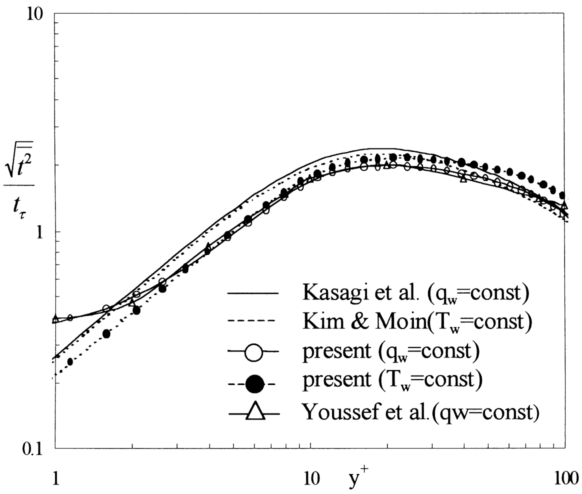

Fig. 2. Temperature variance for the case of constant wall heat flux and constant wall temperature.

$$\frac{\partial \varepsilon_t}{\partial y} = 0 \tag{39}$$

This, as compared with $\varepsilon_{tw} = \alpha(\partial\sqrt{\overline{t^2} - \overline{t^2_w}}/\partial y)^2_w$ given by Youssef et al. [6], would be more convenient actually.

5. Results and discussion

To validate the prediction of the proposed model, we applied it to a thermally and hydrodynamically fully developed turbulent flow in a two-dimensional channel [16,17]. The predicted results with two typical thermal boundary conditions are presented in Figs. 1–7.

The predicted mean-temperature in the turbulent boundary layer is plotted in Fig. 1. The predicted profiles for cases of both uniform wall temperature and uniform wall heat flux are in good agreement with the theoretical distribution, $T^+ = Pry^+$, in the viscous sublayer, and agree with the law of the wall in the logarithmic region.

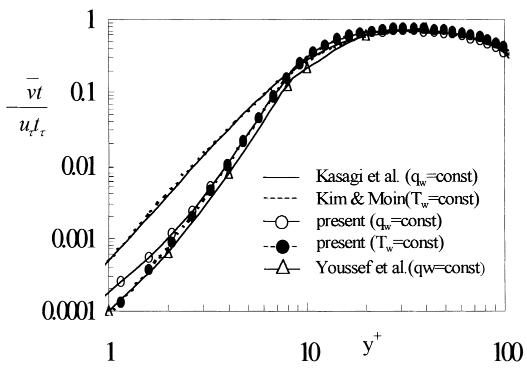

Fig. 3. Near-wall behavior of wall-normal turbulent heat flux.

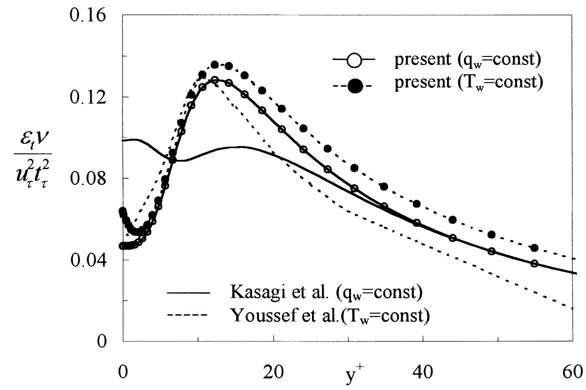

Fig. 4. Dissipation rate of temperature variance for the case of constant wall heat flux and constant wall temperature.

The predicted temperature variance $\overline{t^2}$ in the thermal field is illustrated in log–log form in Fig. 2, using the friction temperature to normalize the root-mean-square temperature variance. The predicted $\overline{t^2}$ undergoes a sharp rise nearby $y^+ = 20$ in the near-wall region, which agrees well with the DNS data [16,17]. The predicted values are much similar to the DNS data in the turbulent core region. The predicted results in viscous sublayer for the case of uniform wall heat flux is slightly larger than DNS data of Kasagi et al. [16], but seems similar to the result of Youssef et al. [6].

The predicted wall-normal turbulent heat flux $-\overline{vt}$ is depicted in Fig. 3, which is in good agreement with the

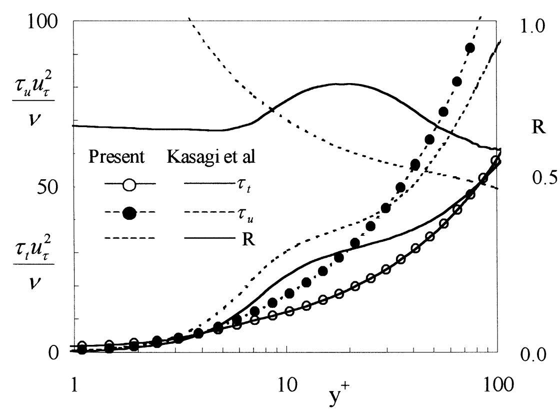

Fig. 5. Comparison of predicted results and DNS data for dissipation timescales and their ratio for the case of constant wall heat flux.

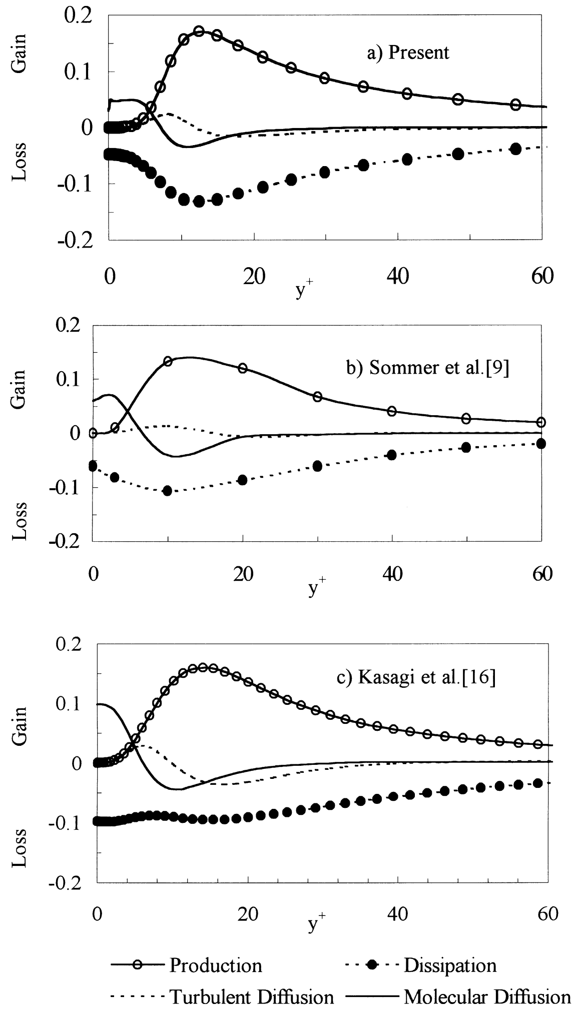

Fig. 6. The budget of temperature variance for the case of constant wall heat flux, (a) present prediction; (b) prediction of Sommer et al. [9]; (c) DNS data [16].

DNS data [16,17] for both thermal boundary conditions except in very near wall region, but agrees well with the result of Youssef et al. [6] in the viscous sublayer.

The prediction of the dissipation rate of the temperature variance ε_t is depicted in Fig. 4. As shown, in the vicinity of the wall, the predicted results for both wall thermal conditions t_w = constant and q_w = constant, deviate from the DNS data [16], which is also shown by that of Youssef et al. [6], but predicted fair well in the region of $y^+ > 20$.

Fig. 5 presents the predicted results of the velocity dissipation timescale τ_u, the temperature dissipation timescale τ_t, and their ratio $R = \tau_t/\tau_u$, for the case of uniform wall heat flux. τ_u would be larger than τ_t over the whole cross section of channel and both timescales increase with increasing y^+. These distributions accord qualitatively well with the DNS data [16]. The difference in the timescale ratio, R, between DNS data and predicted result is observed. The timescale ratio R keeps a finite value at the wall for DNS data [16]. While, the predicted timescale ratio R increases significantly toward the wall and becomes infinite at the wall. This may be consistent with the prediction based on the Taylor series expansion.

The predicted budgets of temperature variance $\overline{t^2}$ and its dissipation rate ε_t are depicted in Figs. 6 and 7, respectively, which are compared with that of [9] and the DNS data [16]. In general, the present model gives a better prediction of the budgets of $\overline{t^2}$ and ε_t.

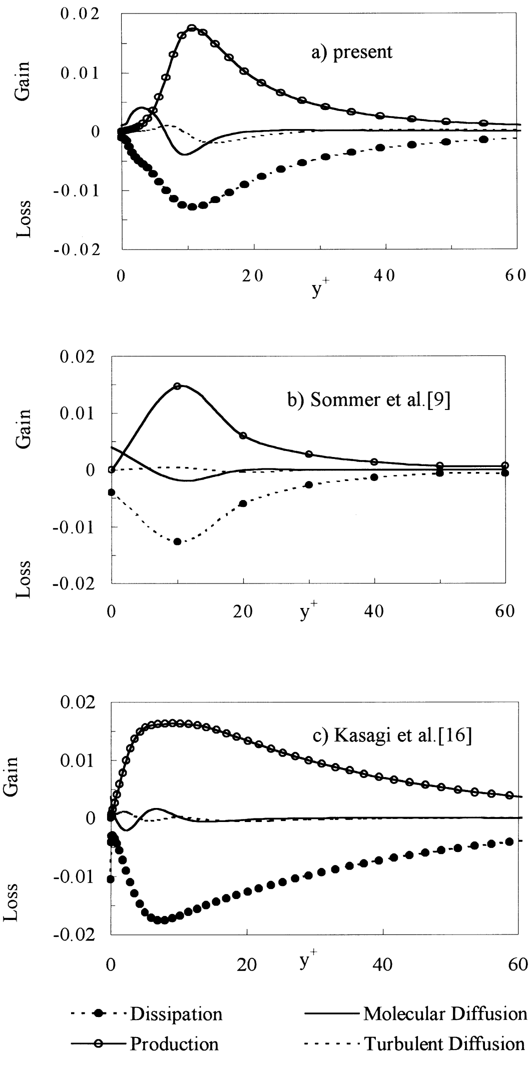

Fig. 7. The budget of the dissipation rate of temperature variance for the case of constant wall heat flux, (a) present prediction; (b) prediction of Sommer et al. [9]; (c) DNS data [16].

6. Conclusions

A new near-wall $\overline{t^2}$–ε_t two-equation heat transfer model has been derived in this study. It is based on the transport equations for temperature variance $\overline{t^2}$ and its dissipation rate ε_t. A modification for modeling ε_t equation is proposed, which use both velocity and thermal timescales and only the thermal production rate to describe the production term in ε_t equation. Near-wall corrections are obtained by analyzing the near-wall limiting behavior of turbulent quantities. Eddy diffusivity for heat is used for relating turbulent heat fluxes to mean temperature field. This involves both velocity and thermal timescales and reproduces the near-wall limiting behavior of normal heat flux under different thermal boundary conditions.

Based on Taylor series expansion, the boundary condition for ε_t under uniform wall heat flux is proposed, which is more convenient than that of previous research for complex heat transfer [6].

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